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Cite as: AIP Conference Proceedings **2176**, 030002 (2019); https://doi.org/10.1063/1.5135126
Published Online: 19 November 2019

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Unidirectional Thermocapillary Flows of a Viscous Incompressible Fluid with the Navier Boundary Condition

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Abstract. A new exact solution for Marangoni shear convection in a flat horizontal layer, induced by taking into account the Navier condition at the lower boundary, is obtained. The study of the behavior of hydrodynamic fields has shown that they have a complex topology and allow the appearance of several zones with reverse flow.

INTRODUCTION

One of the main problems in hydrodynamics is the question of how velocity, temperature, and other fields change as the fluid moves. To answer this question, it is necessary to find a solution to the Navier-Stokes equation, which is an equation for the motion of a viscous fluid, to the heat equation describing the change of the temperature field, and to other basic equations involved in a system defining the viscous fluid model [1-5]. Solutions to systems of partial differential equations are known to depend strongly on boundary conditions [6-17]. This article discusses flow in a flat horizontal layer [8-15]. The constructed exact solution describes the influence of the Marangoni effect [16] and the Navier slip condition [17] on the distribution of hydrodynamic fields. A comparison with previously obtained results is made [8–10, 18-24].

BOUNDARY VALUE PROBLEM FORMULATION

The steady-state shear flow of a viscous incompressible fluid in a flat horizontal layer of thickness h is considered. To describe such flows, we use the following system of heat convection equations in the Boussinesq approximation [1-5]:

$$V_x \frac{\partial V_x}{\partial x} = -\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_x}{\partial z^2}\right);\ \frac{\partial P}{\partial z} = g\beta T;\ V_x \frac{\partial T}{\partial x} = \chi\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right);\ \frac{\partial V_x}{\partial x} = 0. \qquad (1)$$

Here, $V_x = V_x(x,z)$ is the x-component of the velocity vector $\boldsymbol{V}$; $P(x,z)$ is deviation of pressure deviation from hydrostatic, divided by average density; $T(x,z)$ is deviation of temperature from the reference value; ν, χ are the kinematic (molecular) viscosity of the fluid and the coefficient of thermal diffusivity; β is the temperature coefficient of volumetric expansion of the fluid; g is acceleration of gravity.

The solution is sought in the class [7-9],

$$V_x = U(z);\ V_y = V_z = 0; \qquad (2)$$

Mechanics, Resource and Diagnostics of Materials and Structures (MRDMS-2019)
AIP Conf. Proc. 2176, 030002-1–030002-5; https://doi.org/10.1063/1.5135126
Published by AIP Publishing. 978-0-7354-1923-0/$30.00

$$P = P_0(z) + xP_1(z);\ T = T_0(z) + xT_1(z). \quad (3)$$

Class (2)–(3) has the property that the equation of incompressibility (the last equation of system (1)) is identically satisfied for it. Thus, the number of independent equations in this system becomes equal to the number of unknown functions.

Next, we substitute the selected class (2)–(3) into the system of heat convection equations. This will reduce system (1) to a system of ordinary differential equations where each of the incoming functions depends only on the vertical variable z,

$$T_1'' = 0;\ P_1' = g\beta T_1;\ \nu U'' = P_1;\ \chi T_0'' = UT_1,\ P_0' = g\beta T_0. \quad (4)$$

The system of equations (4) is integrated sequentially in the order in which the equations are written out. For the exact solution to be uniquely determined, eight more boundary conditions must be specified.

We will consider the flow of a fluid in the field of gravity assuming that the heat source is set at the upper boundary $z = h$ and that the temperature at the lower boundary $z = 0$ is taken as the reference level,

$$T_0(0) = T_1(0) = 0;\ T_0(h) = 0;\ T_0(h) = A. \quad (5)$$

We also assume that the Navier slip condition is satisfied at the lower boundary and that a uniform pressure (equal to atmospheric pressure) and the tangential stress field caused by the thermocapillary effect are given at the upper boundary,

$$\alpha \left.\frac{\partial U}{\partial z}\right|_{z=0} = U(0);\ P_0(h) = S_0;\ P_1(h) = 0;\ \eta \left.\frac{\partial U}{\partial z}\right|_{z=h} = -\sigma T_1(h). \quad (6)$$

Here, σ and η are the coefficients of temperature-induced surface tension and dynamic viscosity, respectively; α is the slip length.

EQUATION SYSTEM SOLUTION

By considering the boundary conditions (5)–(6) when integrating system (4), we arrive at the following exact solution describing the velocity field, the pressure field and the temperature field:

$$T_1 = \frac{Az}{h};\ P_1 = \frac{Ag\beta}{2h}\left(z^2 - h^2\right);\ U = A\left\{\frac{g\beta}{24\nu h}\left[z^4 - 6h^2z^2 + 8h^3(z+\alpha)\right] - \frac{\sigma}{\eta}(z+\alpha)\right\};$$

$$T_0 = A^2\left\{\frac{\sigma}{12h\chi\eta}(h-z)z\left[h^2 + (h+z)(z+2\alpha)\right] + \right.$$

$$\left. + \frac{g\beta}{5040h^2\nu\chi}\left[5z^7 - 63h^2z^5 + 140h^3z^4 + 280h^3z^3\alpha - 2h^5z(41h + 140\alpha)\right]\right\};$$

$$P_0 = S_0 - \frac{A^2 g\beta\sigma}{120h\eta\chi}(z-h)^2\left(3h^3 + 2hz(2z+5\alpha) + z^2(2z+5\alpha) + h^2(6z+5\alpha)\right) +$$

$$+ \frac{A^2g^2\beta^2}{40320h^2\nu\chi}(z-h)^2\left[183h^6 - 69h^2z^4 + 10hz^5 + 5z^6 + \right.$$

$$\left. + h^4z(221z + 1120\alpha) + h^5(366z + 560\alpha) + 4h^3z^2(19z + 140\alpha)\right]. \quad (7)$$

INVESTIGATION OF THE SOLUTION

Note that, when the longitudinal temperature gradient A is zero, the exact solution (7) is trivial,

$$T_1 = T_0 = 0\ ;\ U = 0\ ;\ P_1 = 0\ ;\ P_0 = S_0\,.$$

If $A\neq 0$, the value of this gradient does not affect the location of the zero points of the components T_0, T_1, U, and P_1, affecting only their values. In this case, the behavior of hydrodynamic fields (7) also depends on the parameter α.

All the three hydrodynamic fields (the velocity field, the pressure field, and the temperature field), according to expressions (7), are determined by the interaction of thermogravitational and thermocapillary flows.

Obviously, the longitudinal gradients T_1 and P_1 vanish only on the boundary of the layer under study $[0,h]$. Therefore, the corresponding fields do not allow stratification. The analysis of the properties of the polynomials determining the exact solution (7) demonstrates that the velocity U can have only one stagnant (zero) point since the slip length α can take only positive values (Fig. 1). The temperature field and the pressure field can also stratify, and the number of their stratification points does not exceed one (Fig. 2.3).

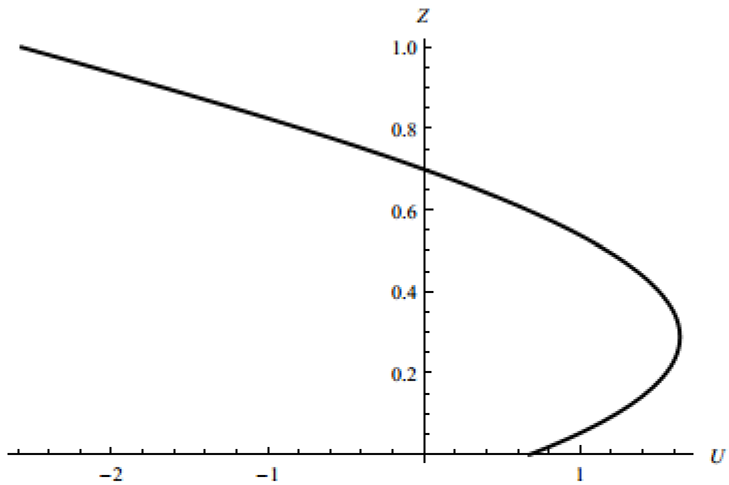

FIGURE 1. The behavior of the velocity

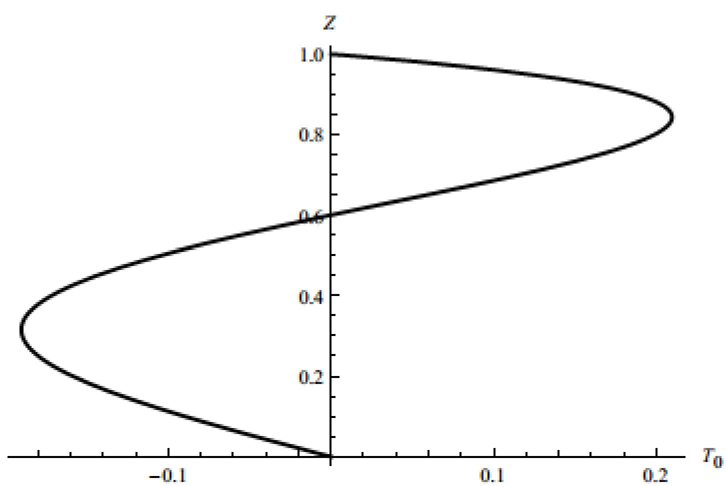

FIGURE 2. The behavior of the background temperature

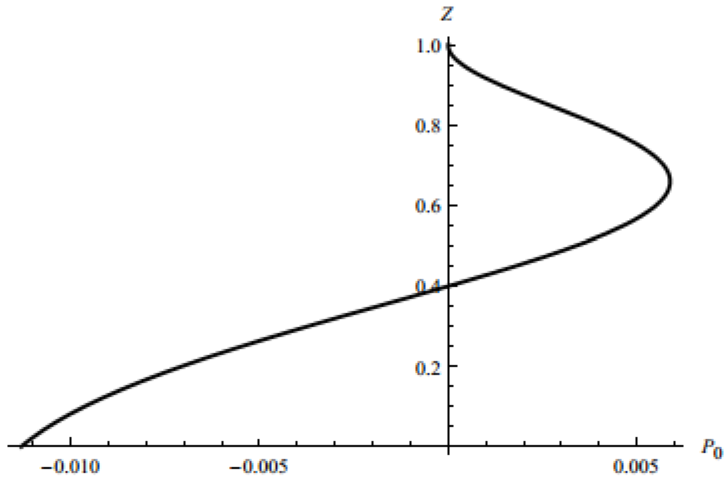

FIGURE 3. The behavior of the background pressure

CONCLUSION

A new solution describing the influence of the Marangoni effect on the flow of a viscous incompressible fluid has been proposed. This solution is determined by high-order polynomials. However, in view of the fact that the Navier slip condition is satisfied at the lower boundary, the obtained exact solution admits no more than one point for any of the main hydrodynamic fields.

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